

The radiative decays of 0^{++} and 1^{+-} heavy mesons

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Abstract

The radiative decay is believed to be an ideal lab to study hadronic structure of newly observed resonances because the reactions are governed by only the electromagnetic interaction (tree level). However, to obtain correct theoretical values, one has to properly deal with the non-perturbative QCD effects in the wavefunction and hadronization. In this work we derive the formulas for the radiative decays of 0^{++} and 1^{+-} heavy mesons in the light front quark model (LFQM). Because $\mathcal{B}(\chi_{c0} \rightarrow J/\psi\gamma)$ is well measured, the theoretical evaluation of the transition rate can be used to test our approach. Within this theoretical framework, the width of $\chi_{b0} \rightarrow \Upsilon(1S)\gamma$ is evaluated. The formulas can be applied to identify the inner structures of new resonances, for example the isospin of $h_{c(b)}$ and the structure of $D_s(2317)$, via processes $h_c \rightarrow \eta_c\gamma$, $h_b \rightarrow \eta_b\gamma$ and $D_s(2317) \rightarrow D_s^* + \gamma$.

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I. INTRODUCTION

In the field of heavy hadrons, it is noticed that some experimental observations obviously deviate from our theoretical predictions, so definitely, such “anomalies” need to be clarified. Supposing the experimental measurements are right, there must be some loopholes in our present understanding of the nature, either there exists a contribution from new physics beyond the standard model (SM), or the concerned hadrons have exotic structure such as hybrid, multi-quark etc. It is believed that radiative decays on this aspect provide an ideal lab to testify the hadron structures and as well help to identify the quantum numbers for a newly observed resonances. Even though, the reaction mechanism for radiative decays is free of strong interaction (at tree level), the non-perturbative QCD effects are still involved in the hadronization of emerging hadrons, as well as the wavefunction of the parent hadron. To search for new physics or possible exotic components of the hadron, one must thoroughly study how the non-perturbative QCD affects the decay rates in a reasonable theoretical framework. In this work, we are going to derive the formulas for the radiative decays of 0^{++} and 1^{+-} heavy mesons which are supposed to have regular $q\bar{q}'$ structures, in the light front quark model (LFQM). Through a comparison of the theoretical predictions with data, the consistency degree would reveal if the concerned hadron possesses the regular $q\bar{q}'$ structure or has an exotic component.

During recent years several new particles such as $D_s(2317)$ [1], $D_s(2460)$ [2, 3], $X(3872)$ [4], $X(3940)$ [5] and $Y(3940)$ [6] have been observed in experiments and re-confirmed as new resonances. However it is not easy to identify their inner structures, i.e. if the constituents of the resonances are indeed just q and \bar{q}' or with something else (gluon or $q\bar{q}$ pair etc.). Some of them are speculated as exotic states. The reasons may be twofold: one is their peculiar decay modes whereas the other is that the theoretical expectations on their excited states are not well consistent with data or even missing. For example when $D_s(2317)$ was announced it was considered as a molecular state or a tetraquark, however it may just be a regular p -wave $c\bar{s}$ meson with $J^P = 0^+$ as it appears in particle data book[7]. Because radiative decay is fully governed by the electromagnetic interaction, the reaction mechanism is relatively simple and the emitted photon can be well measured in experiment, it may offer a good opportunity to justify the quantum numbers and constituent structure of the newly observed resonance. Namely, one may check whether with the simple $q\bar{q}'$ assignment the theoretical prediction agrees with data.

For p -wave particles there are three degenerate states 0^{++} , 1^{++} and 2^{++} with the total intrinsic spin $S = 1$ and one singlet 1^{+-} with $S = 0$. It is well known that χ_{c0}, χ_{c1} and χ_{c2} and h_c are triplets and singlet p -wave charmonia respectively.

In this work we will calculate the rates of $0^{++} \rightarrow 1^{--}\gamma$ and $1^{+-} \rightarrow 0^{-+}\gamma$ in LFQM [8–15] which has been successfully applied to evaluate rates of semileptonic and non-leptonic decays of s -wave heavy mesons. For the p -wave mesons, the wave functions of 0^{++} , 1^{--} , 1^{+-} and 0^{-+} have been constructed and the leading Feynman diagrams are simple, with them we are able to calculate the corresponding transition amplitudes.

Since the branching ratio of $\chi_{c0} \rightarrow J/\psi\gamma$ is well measured, we first calculate $B(\chi_{c0} \rightarrow$

$J/\psi\gamma$) in LFQM to fix the model parameters and check the validity degree of this approach, then with the formulas for the transition $0^{++} \rightarrow 1^{--}\gamma$ we estimate the rates of $\chi_{b0} \rightarrow \Upsilon(1S)\gamma$ and $D_s(2317) \rightarrow D_s^* + \gamma$. These formulas can also be applied to study decays of other 0^{++} states. Recently the spin singlets h_c and h_b attract intensive interests of both experimentalists and theorists[16–18]. By the formulas for $1^{+-} \rightarrow 0^{-+}\gamma$ we calculate the widths of $h_c \rightarrow \eta_c\gamma$ and $h_b \rightarrow \eta_b\gamma$. The results can be compared with the data which will be available soon at the BES II and future B-factory. This comparison definitely assures us if they are pure p -wave heavy quarkonia.

After the introduction we derive the formulas for the transition $0^{++} \rightarrow 1^{--}\gamma$ and $1^{+-} \rightarrow 0^{-+}\gamma$ in section II. Then in Sec. III, we numerically evaluate the decay widths of $\chi_{c0} \rightarrow J/\psi\gamma$, $\chi_{b0} \rightarrow \Upsilon(1S)\gamma$, $D_s(2317) \rightarrow D_s^* + \gamma$, $h_c \rightarrow \eta_c\gamma$ and $h_b \rightarrow \eta_b\gamma$ and make some discussions. In the last section we give a brief summary and discussion. Some notations and definitions of relevant quantities are collected in the attached appendix.

II. THE FORMULA FOR THE DECAYS $0^{++} \rightarrow 1^{--}\gamma$ AND $1^{+-} \rightarrow 0^{-+}\gamma$

In Ref.[19] Chung studied various transitions and derived the corresponding amplitudes, for example, he determined the amplitude structure of $b_1(1235) \rightarrow \omega\pi$. For transition $1^{+-} \rightarrow 0^{-+}\gamma$ [19] a photon exists in the final states, thus the gauge invariance demands the transition amplitude to be in the form

$$A = F * (g_{\mu\nu} - \frac{k_\mu q_\nu}{k \cdot q}) \varepsilon^\mu \varepsilon'^\nu, \quad (1)$$

where q and k represent the momenta of the photon and daughter meson, ε' and ε are the polarizations of the vector-meson and photon respectively, and F is the form factor which is what we are going to derive and numerically compute in this work. Though the $0^{++} \rightarrow 1^{--}1^{--}$ was not discussed in Ref.[19], its amplitude structure is the same as Eq.(1) and it can be seen from analyzing the J^P characters of the involved mesons.

Below, following the schemes given in literature, we will calculate the transition amplitudes of interest in LFQM.

A. the decay of $0^{++} \rightarrow 1^{--}\gamma$

The vertex functions of 0^{++} and 1^{--} are respectively [12]

$$-iH_S \quad (2)$$

$$iH_V[\gamma_\mu - \frac{1}{W_V}(p_a - p_b)_\mu] \quad (3)$$

where H_S , H_V and W_V are defined in Ref.[12] and p_a and p_b are the momenta of constituents of the corresponding meson.

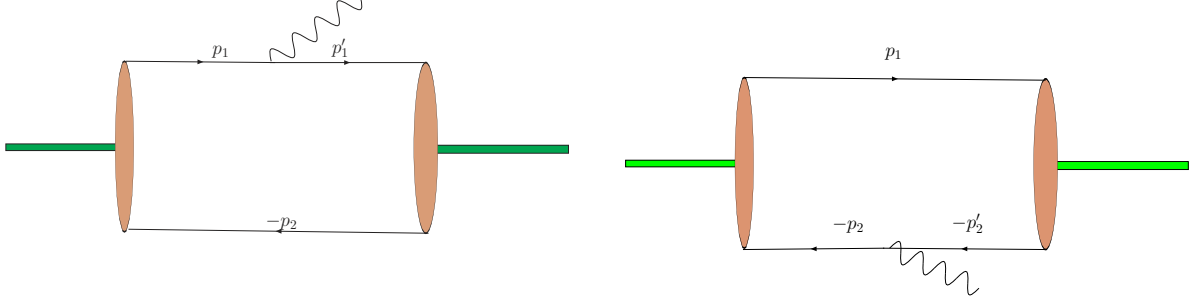


FIG. 1: Feynman diagrams for radiative decay.

The transition amplitude corresponding to the left diagram of Fig.1 is written as

$$\mathcal{A}^a = iee_1 \frac{N_c}{(2\pi)^4} \int d^4 p_1 \frac{H_S H_V}{N_1 N'_1 N_2} s_{\mu\nu}^a \varepsilon'^\nu \varepsilon^\mu \quad (4)$$

where

$$s_{\mu\nu}^a = \text{Tr}\{(-\not{p}_2 + m_2)[\gamma_\nu - \frac{(p'_1 - p_2)_\nu}{W_V}](\not{p}'_1 + m_1)\gamma_\mu(\not{p}_1 + m_1)\},$$

$N_1 = p_1^2 - m_1^2 + i\epsilon$, $N'_1 = p'^2_1 - m_1^2 + i\epsilon$, $N_2 = p_2^2 - m_2^2 + i\epsilon$, $ee_{1(2)}$ is the electric charge of the quark of u- or d-types. In the light front frame, p_i is decomposed as $(p_i^-, p_i^+, p_{i\perp})$. Integrating out p_1^- with the methods given in Ref.[11] one has

$$\int d^4 p_1 \frac{H_S H_V}{N_1 N'_1 N_2} s_{\mu\nu}^a \varepsilon'^\nu \varepsilon^\mu \rightarrow -i\pi \int dx_1 d^2 p_\perp \frac{h_S h_V}{x_2 \hat{N}_1 \hat{N}'_1} \hat{s}_{\mu\nu}^a \hat{\varepsilon}'^\nu \hat{\varepsilon}^\mu, \quad (5)$$

with

$$\begin{aligned} h_S &= (M^2 - M_0^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2} \tilde{M}_0} \frac{\tilde{M}_0^2}{2\sqrt{3} M_0} \frac{\sqrt{2}}{\beta} \phi(nS), \\ h_V &= (M'^2 - M'^2_0) \sqrt{\frac{x'_1 x'_2}{N_c}} \frac{1}{\sqrt{2} \tilde{M}'_0} \phi'(nS), \\ \hat{N}_1^{(\prime)} &= x_1^{(\prime)} (M^{(\prime)2} - M_0^{(\prime)2}), \end{aligned}$$

where M and M' represent the masses of decaying and produced mesons and the relation between $\varepsilon(\varepsilon')$ and $\hat{\varepsilon}(\hat{\varepsilon}')$ can be found in the appendix of Ref.[12].

To include the contributions from the zero mode, $p_{1\mu}$, $p_{1\nu}$, $p_{1\mu} p_{1\nu}$ and W_V in $s_{\mu\nu}^a$ should be replaced by appropriate expressions as discussed in Ref.[12], for example

$$\begin{aligned} W_V &\rightarrow w_V = M_0 + m_1 + m_2 \\ p_{1\mu} &\rightarrow \frac{x_1}{2} \mathcal{P}_\mu + (\frac{x}{2} - \frac{p_\perp q_\perp}{q^2}) q_\mu, \\ &\dots\dots \end{aligned} \quad (6)$$

with $\mathcal{P} = p + k$ and p is the momentum of decaying meson.

The definitions of M_0 and \tilde{M}_0 are presented in the appendix, as more details about the derivations and relevant notations can be found in Ref.[12]. In this framework, $\hat{s}_{\mu\nu}^a$ replaces $s_{\mu\nu}^a$ and is written into the form

$$\hat{s}_{\mu\nu}^a = F_1 g_{\mu\nu} + F_2 \frac{k_\mu q_\nu}{k \cdot q}, \quad (7)$$

with

$$\begin{aligned} F_1 = & -2 \left[m_1 \left(M^2 - M'^2 - \hat{N}_1 + \hat{N}_1' \right) + m_2 \left(\hat{N}_1 + \hat{N}_1' - q^2 \right) \right] - \\ & \frac{4 A_{12} \left(2 m_1^2 + 4 m_1 m_2 + 2 m_2^2 - M^2 - M'^2 + q^2 - 2 m_1 w_V - 2 m_2 w_V + 2 Z_2 \right)}{w_V}, \\ F_2 = & \frac{4 k \cdot q A_2^{(1)} \left(-\hat{N}_1 - \hat{N}_1' + q^2 \right)}{w_V} + \frac{4 k \cdot q \left(\hat{N}_1 + \hat{N}_1' - q^2 + m_1 w_V \right)}{w_V} + \\ & \frac{4 k \cdot q A_2^{(2)} \left\{ -2 \left[2 (m_1 + m_2)^2 - M^2 - M'^2 + q^2 \right] + 4 m_1 w_V + 4 m_2 w_V \right\}}{w_V} + \\ & \frac{4 k \cdot q A_3^{(2)} \left\{ -2 \left[2 (m_1 + m_2)^2 - M^2 - M'^2 + q^2 \right] + 4 m_1 w_V + 4 m_2 w_V \right\}}{w_V} + \\ & \frac{4 k \cdot q A_1^{(1)} \left[4 (m_1 + m_2)^2 - 2 M^2 - 2 M'^2 - \hat{N}_1 - \hat{N}_1' + 3 q^2 - 2 (3 m_1 + m_2) w_V \right]}{w_V} \end{aligned} \quad (8)$$

and $A_i^{(j)} (i = 1 \sim 4, j = 1 \sim 4)$ are presented in the attached appendix.

We define the form factors as following

$$\begin{aligned} f_1(m_1, m_2, e_1) &= \frac{ee_1}{32\pi^3} \int dx_2 d^2 p_\perp \frac{F_1 \phi \phi' \tilde{M}_0}{\sqrt{6} \beta M_0 \tilde{M}'_{0x_1}}, \\ f_2(m_1, m_2, e_1) &= \frac{ee_1}{32\pi^3} \int dx_2 d^2 p_\perp \frac{F_2 \phi \phi' \tilde{M}_0}{\sqrt{6} \beta M_0 \tilde{M}'_{0x_1}}, \end{aligned} \quad (9)$$

where ϕ and ϕ' are respectively the wavefunctions of the initial and final mesons. These form factors will be numerically evaluated in next section.

With these form factors the amplitude corresponding to the left diagram of Fig.1 is obtained as

$$\mathcal{A}^a = f_1(m_1, m_2, e_1) \varepsilon \cdot \varepsilon' + f_2(m_1, m_2, e_1) \frac{k \cdot \varepsilon' q \cdot \varepsilon}{k \cdot q}. \quad (10)$$

The right diagram is just the charge conjugation of the left one of Fig.1, so that one can immediately write it down

$$\mathcal{A}^b = f_1(m_2, m_1, e_2) \varepsilon \cdot \varepsilon' + f_2(m_2, m_1, e_2) \frac{k \cdot \varepsilon' q \cdot \varepsilon}{k \cdot q}. \quad (11)$$

The total amplitude is simply a sum of \mathcal{A}^a and \mathcal{A}^b :

$$\mathcal{A} = \mathcal{A}^a + \mathcal{A}^b. \quad (12)$$

Comparing Eq. (1) with Eq. (12) we determine the full form factor as

$$F = f_1(m_1, m_2, e_1) + f_1(m_2, m_1, e_2) = -[f_2(m_1, m_2, e_1) + f_2(m_2, m_1, e_2)]. \quad (13)$$

In principle one can calculate $f_1 = f_1(m_1, m_2, e_1) + f_1(m_2, m_1, e_2)$ and $f_2 = f_2(m_1, m_2, e_1) + f_2(m_2, m_1, e_2)$ separately and fix the form factor, then go on obtaining the decay rate.

B. the decay rate of $1^{+-} \rightarrow 0^{-+}\gamma$

The vertex functions of 0^{-+} and 1^{+-} are presented in Ref.[12], they are respectively

$$H_P \gamma_5; \quad (14)$$

$$-iH_{1A}[\frac{1}{W_{1A}}(p_a - p_b)_\mu] \gamma_5, \quad (15)$$

where H_P , H_{1A} and W_{1A} are some relevant functions which are slightly lengthy and can be found in Ref.[12], for saving space, we do not repeat them here.

The transition amplitude corresponding to the left diagram of Fig.1 is written as

$$\mathcal{A}^a = ee_1 \frac{N_c}{(2\pi)^4} \int d^4 p_1 \frac{H_P H_{1A}}{N_1 N'_1 N_2} s_{\mu\nu}^a \epsilon^\nu \epsilon^\mu \quad (16)$$

where

$$s_{\mu\nu}^a = \text{Tr}\{(-\not{p}_2 + m_2)[\frac{(p'_1 - p_2)_\nu}{W_{1A}} \gamma_5](\not{p}'_1 + m_1)\gamma_\mu(\not{p}_1 + m_1)\gamma_5\}.$$

Integrating out p_1^- we have

$$\int d^4 p_1 \frac{H_P H_{1A}}{N_1 N'_1 N_2} s_{\mu\nu}^a \epsilon^\nu \epsilon^\mu \rightarrow -i\pi \int dx_1 d^2 p_\perp \frac{h_P h_{1A}}{x_2 \hat{N}_1 \hat{N}'_1} \hat{s}_{\mu\nu}^a \hat{\epsilon}'^\nu \hat{\epsilon}^\mu, \quad (17)$$

where

$$h_{1A} = (M^2 - M_0^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2} \tilde{M}_0} \frac{\sqrt{2}}{\beta} \phi(nS),$$

$$h_P = (M'^2 - M_0'^2) \sqrt{\frac{x'_1 x'_2}{N_c}} \frac{1}{\sqrt{2} \tilde{M}'_0} \phi'(nS).$$

With a replacement similar to Eq.(6), $\hat{s}_{\mu\nu}^a$ is re-written into the form of Eq. (7) with

$$\begin{aligned} F_1 = & A_1^{(2)} \left(\frac{-8 m_1^2}{w_{1A}} + \frac{16 m_1 m_2}{w_{1A}} - \frac{8 m_2^2}{w_{1A}} + \frac{4 M^2}{w_{1A}} + \frac{4 M'^2}{w_{1A}} - \frac{4 q^2}{w_{1A}} - \frac{8 Z_2}{w_{1A}} \right) \\ F_2 = & 2 k \cdot q [A_1^{(1)} \left(\frac{2 \hat{N}_1}{w_{1A}} + \frac{2 \hat{N}'_1}{w_{1A}} - \frac{2 q^2}{w_{1A}} \right) + A_2^{(1)} \left(\frac{-2 \hat{N}_1}{w_{1A}} - \frac{2 \hat{N}'_1}{w_{1A}} + \frac{2 q^2}{w_{1A}} \right) + \\ & A_2^{(2)} \left(\frac{8 m_1^2}{w_{1A}} - \frac{16 m_1 m_2}{w_{1A}} + \frac{8 m_2^2}{w_{1A}} - \frac{4 M^2}{w_{1A}} - \frac{4 M'^2}{w_{1A}} + \frac{4 q^2}{w_{1A}} \right) + \\ & A_3^{(2)} \left(\frac{-8 m_1^2}{w_{1A}} + \frac{16 m_1 m_2}{w_{1A}} - \frac{8 m_2^2}{w_{1A}} + \frac{4 M^2}{w_{1A}} + \frac{4 M'^2}{w_{1A}} - \frac{4 q^2}{w_{1A}} \right)]. \end{aligned} \quad (18)$$

TABLE I: the form factors f_1 and f_2

decay mode	f_1	f_2	width(keV)	$\mathcal{BR}(\text{the})$	$\mathcal{BR}(\text{exp})$
$\chi_{c0} \rightarrow J/\psi\gamma$	-2.83 ± 1.37	0.89 ± 0.09	152 ± 31	$(1.46 \pm 0.31)\%$	$(1.17 \pm 0.08)\% [7]$
$\chi_{b0} \rightarrow \Upsilon(1S)\gamma$	-0.50 ± 3.01	0.82 ± 0.08	21.3 ± 4.7	-	$(1.76 \pm 0.35)\% [7]$
$D_s(2317) \rightarrow D_s^*\gamma$	-1.11 ± 0.22	0.25 ± 0.03	17.1 ± 3.9	$> (0.45 \pm 0.11)\%$	-
$h_c \rightarrow \eta_c\gamma$	-2.36 ± 0.34	2.47 ± 0.18	685 ± 89	$> (69 \pm 9)\%$	$(51 \pm 6)\% [7]$
$h_b \rightarrow \eta_b\gamma$	-2.40 ± 0.31	1.78 ± 0.20	36.9 ± 8.7	-	$(49.2 \pm 5.7_{-3.3}^{+5.6})\% [20]$

TABLE II: Predictions made in terms of various approaches and experimental data (if available) (in units of keV)

decay mode	[24]	[25]	our results	exp
$\chi_{c0} \rightarrow J/\psi\gamma$	121	input	152 ± 31	$122 \pm 12 [7]$
$\chi_{b0} \rightarrow \Upsilon(1S)\gamma$	29.9	85 ± 4	21.3 ± 4.7	-
$h_c \rightarrow \eta_c\gamma$	560	634 ± 32	685 ± 89	-
$h_b \rightarrow \eta_b\gamma$	52.6	271 ± 14	36.9 ± 8.7	-

III. APPLYING THE APPROACH TO ANALYZE RADIATIVE DECAYS

In this section we first test the formula by comparing our theoretical evaluation on the rate of a well measured decay mode with the data and confirm its validity, then apply it to predict the rates of radiative decays of 0^{++} and 1^{+-} which will be experimentally measured soon. We select transition $\chi_{c0} \rightarrow J/\psi\gamma$ as a probe to check the approach since its branching ratio is well measured. Setting $m_c = 1.4\text{GeV}$ and the model parameters $\beta_{\psi(\chi_c)} = 0.631\text{GeV}[21]^1$, we get the form factors f_1 and f_2 for the transition $\chi_{c0} \rightarrow J/\psi\gamma$ which are presented in Tab. I. One can immediately notice that f_1 is sensitive to the variation of the parameters, especially m_c , but f_2 is insensitive. The problem originates from the fact that $\hat{N}_1^{(r)}$ is proportional to a cancelation of two large numbers. Even though $\hat{N}_1^{(r)}$ resides in both F_1 and F_2 , its contribution is suppressed by w_V or w_{1A} in F_2 , so that F_2 is insensitive to the parameters. For reducing uncertainties of our theoretical computations, we will calculate F in terms of the relation between f_2 and F in Eq.(13). Our theoretical estimate of the width of $\chi_{c0} \rightarrow J/\psi\gamma$ is $(152 \pm 31)\text{keV}$ and the corresponding branching ratio is $(1.46 \pm 0.31)\%$ which is consistent with the data $(1.17 \pm 0.08)\%[7]$ within a tolerable error range.

Then we study the transition $\chi_{b0} \rightarrow \Upsilon(1S)\gamma$ using the parameter $m_b = 4.8\text{GeV}$ and $\beta_{\Upsilon(\chi_b)} = 1.288\text{GeV}$ which were fixed in Ref.[22] by fitting other well measured channels. Our estimate of $\Gamma(\chi_{b0} \rightarrow \Upsilon(1S)\gamma)$ is $(21.3 \pm 4.7)\text{keV}$. With the branching ratio $(1.76 \pm 0.35)\%[7]$, the total width of χ_{b0} is estimated to be $(1.21 \pm 0.36)\text{MeV}$.

Supposing $D_s(2317)$ to be a regular bound state of $c\bar{s}$ and setting $m_s = 0.37\text{GeV}$, $\beta_{D_s} =$

¹ We vary the parameters within a $\pm 10\%$ range to estimate the corresponding errors.

0.592GeV[23], we can calculate the rate of $D_s(2317) \rightarrow D_s^* + \gamma$ using the above formulas. We obtain its partial width as $(17.1 \pm 3.9)\text{keV}$. Namely, if $D_s(2317)$ is a 0^{++} regular meson, possessing the p -wave $c\bar{s}$ structure, our numerical prediction should be consistent with the data which will be available at BES II or B-factory soon, therefore the consistency degree with data can help to confirm or negate its assignment $c\bar{s}$.

In fact these formulas can be applied to study radiative decays of other heavy mesons such D_0 , B_0 , B_{s0} and B_{c0} .

Once h_c and h_b were experimentally measured, their special behaviors draw intensive interests of theorists. The main focus is if they are regular quarkonia or have exotic components. Using the formulas for $1^{+-} \rightarrow 0^{-+}\gamma$, we calculate the branching ratios of $h_c \rightarrow \eta_c\gamma$ and $h_b \rightarrow \eta_b\gamma$ and the numerical result are presented in Tab. I, which can be used to analyze their characters. In terms of the measured upper limit of the total width of h_c we estimate the branching ratio of $\mathcal{B}(h_c \rightarrow \eta_c\gamma) > (69 \pm 9)\%$ which is a little larger than the data $(51 \pm 6)\%$. If h_c (h_b) is a pure charmonium (bottomonium) our estimate indicates that its total width should be around 1 MeV (80keV). To further identify the inner structures of h_c and h_b , more precise experiments are needed.

It is noted that the masses of initial and final mesons we used in our numerical computations are taken from Ref.[7] except the mass of η_b which is 9402.4MeV[20]. Some parallel researches on these decays can be found in Ref.[24, 25] and a comparison of their results with ours is made and listed in Tab. II. Because of large errors in the inputs for our numerical computations, our predictions are of relatively large uncertainties. To further testify the model or constrain the model parameters, much more precise experiments are needed.

IV. SUMMARY

In this work we derive the formulas for the radiative decay of 0^{++} and 1^{+-} heavy mesons and numerically compute the rates. We formulate the transition matrix elements and extract the form factors in the LFQM. To check the validity degree of this approach where the model parameters were fixed by fitting other physical processes in previous works, we calculate the decay widths of $\chi_{c0} \rightarrow J/\psi\gamma$ and compare it with data. Considering both theoretical and measurement uncertainties, the theoretical result on the branching ratio of $\chi_{c0} \rightarrow J/\psi\gamma$ $(1.46 \pm 0.31)\%$ is satisfactorily consistent with data $(1.17 \pm 0.08)\%$. With the same scenario, we predict the width of $\chi_{b0} \rightarrow \Upsilon(1S)\gamma$ to be about $(21.3 \pm 4.7)\text{keV}$ and the width of $D_s(2317) \rightarrow D_s^* + \gamma$ to be $(17.1 \pm 3.9)\text{keV}$. Comparing those results with the data which will be available at BES and B-factory, one can further confirm the inner structure of χ_{b0} and $D_s(2317)$.

In terms of the data $B(\chi_{b0} \rightarrow \Upsilon(1S)\gamma)$ we estimate the total width of χ_{b0} as $(1.21 \pm 0.36)\text{MeV}$ which would be easier to be experimentally checked. As for the 1^{+-} decay, we evaluate the decay rates of $h_c \rightarrow \eta_c\gamma$ and $h_b \rightarrow \eta_b\gamma$ which are $(685 \pm 89)\text{keV}$ and $(36.9 \pm 8.7)\text{keV}$ respectively. These formulas deduced in this work can also be applied to study radiative decays of other 0^{++} and 1^{+-} particles.

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Appendix A: Model description

In the conventional light-front model, a meson containing a quark q_1 and an antiquark \bar{q}_2 with its total momentum P and spin J can be expressed[11]

$$|\mathcal{M}(P^{2S+1}, L_J, J_z)\rangle = \int \{d^3p_1\} \{d^3p_2\} 2(2\pi)^3 \delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2) \times \sum_{\lambda_1, \lambda_2} \Psi_{LS}^{JJ_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) |q_1(p_1, \lambda_1) \bar{q}_2(p_2, \lambda_2)\rangle, \quad (\text{A1})$$

where $\Psi_{LS}^{JJ_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2)$ is wave function in momentum-space, λ_1 and λ_2 denote helicities, p_1, p_2 are the on-mass-shell light-front momenta defined by

$$\tilde{p}_i = (p_i^+, p_{i\perp}), \quad p_{i\perp} = (p_i^1, p_i^2), \quad p_i^- = \frac{m^2 + p_{i\perp}^2}{p_i^+}, \quad (\text{A2})$$

and

$$\{d^3p_i\} \equiv \frac{dp_i^+ d^2p_{i\perp}}{2(2\pi)^3}, \quad \delta^3(\tilde{p}_i) = \delta(p_i^+) \delta^2(p_{i\perp}). \quad (\text{A3})$$

From the eigen-equation[26]

$$\mathcal{H}_{LF} |\mathcal{M}(P^{2S+1}, L_J, J_z)\rangle = \frac{M^2 + P_\perp^2}{P_i^+} |\mathcal{M}(P^{2S+1}, L_J, J_z)\rangle, \quad (\text{A4})$$

one can deduce a light-front Bethe-Salpeter equation

$$(M^2 - M_0^2) \phi(x, p_\perp) = \int \frac{dx' d^2p'_\perp}{2(2\pi)^3} V_{eff} \phi(x', p'_\perp). \quad (\text{A5})$$

In principle by solving Eq.(A5) one can obtain the momentum distribution amplitude $\phi(x, p_\perp)$. However the two-body interaction kernel V_{eff} is complicate so some phenomenological amplitudes are chosen in practicable theoretical calculations. In this work we use the Gaussian-type amplitudes, for example

$$\phi(1S) = 4 \left(\frac{\pi}{\beta^2}\right)^{3/4} \sqrt{\frac{dp_z}{dx_2}} \exp\left(-\frac{p_z^2 + p_\perp^2}{2\beta^2}\right), \quad (\text{A6})$$

where β is the model parameter to be fixed by calculating the decay constant. A parallel approach about LFQM was done in Refs.[27, 28] where the Gaussian-type amplitude is adopted as a trial function for the variational computations of the QCD-motivated effective Hamiltonian.

Appendix B: Notations

Here we list some variables and notations appearing in the context. The incoming meson in Fig. 1 has momentum $P = p_1 + p_2$ where p_1 and p_2 are the momenta of the off-shell quark and antiquark and

$$\begin{aligned} p_1^+ &= x_1 P^+, & p_2^+ &= x_2 P^+, \\ p_{1\perp} &= x_1 P_\perp + p_\perp, & p_{2\perp} &= x_2 P_\perp - p_\perp, \end{aligned} \quad (\text{B1})$$

with x_i and p_\perp are internal variables and $x_1 + x_2 = 1$.

The variables M_0 , \tilde{M}_0 and \hat{N}_1 are defined as

$$\begin{aligned} M_0^2 &= \frac{p_\perp^2 + m_1^2}{x_1} + \frac{p_\perp^2 + m_2^2}{x_2}, \\ \tilde{M}_0 &= \sqrt{M_0^2 - (m_1 - m_2)^2}. \end{aligned} \quad (\text{B2})$$

with $p_z = \frac{x_2 M_0}{2} - \frac{m_2^2 + p_\perp^2}{2x_2 M_0}$.

$A_{ij}(i = 1 \sim 4, j = 1 \sim 4)$ are

$$\begin{aligned} A_1^{(1)} &= \frac{x_1}{2}, \quad A_2^{(1)} = A_1^{(1)} - \frac{p_\perp \cdot q_\perp}{q^2}, \quad A_1^{(2)} = -p_\perp^2 - \frac{(p_\perp \cdot q_\perp)^2}{q^2}, \\ A_2^{(2)} &= (A_1^{(1)})^2, \quad A_3^{(2)} = A_1^{(2)} A_2^{(2)}, \quad A_4^{(2)} = (A_2^{(1)})^2 - \frac{A_1^{(2)}}{q^2}, \\ A_1^{(3)} &= A_1^{(1)} A_{12}, \quad A_2^{(3)} = A_2^{(1)} A_1^{(2)}, \quad A_3^{(3)} = A_1^{(1)} A_2^{(2)}, \\ A_4^{(3)} &= A_2^{(1)} A_2^{(2)}, \quad A_1^{(4)} = \frac{(A_1^{(2)})^2}{3}, \quad A_2^{(4)} = A_1^{(1)} A_1^{(3)}, \\ A_3^{(4)} &= A_1^{(1)} A_2^{(3)}, \quad A_4^{(4)} = A_2^{(1)} A_1^{(3)} - \frac{A_1^{(4)}}{q^2}. \end{aligned} \quad (\text{B3})$$

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